Numerical Simulation of Covid-19 Mathematical Modelling with Optimal Control in Indonesia

Nur Ilmayasinta[1(✉)] and Asmianto[2]

[1] Mathematics Education Department, Universitas Islam Lamongan, Lamongan, Indonesia
nurilma@unisla.ac.id

[2] Mathematics Department, Universitas Negeri Malang, Malang, Indonesia

Abstract. The mathematical model of COVID-19 considered in this study is the SEIR model which is defined by four ordinary differential equations that describe the number of susceptible, infected, infected and cured individuals by applying optimal control theory in the form of treatment and quarantine. To characterize the optimal control in the COVID-19 seir mathematical model, the Pontryagin maximum principle is used. The purpose of this study was to reduce the number of susceptible, infected and infected individuals and increase the number of recovered individual populations. The covid-19 mathematical model with optimal control is solved using the Runge-kutta order 4 method and the results are represented graphically. The results obtained from the simulations carried out show that optimal control can work well on the Covid-19 mathematical model that has been formed with the data used being actual data on Covid-19 cases in Indonesia.

Keywords: Covid-19 · mathematical model · optimal control · SEIR model · PMP method

1 Introduction

An infectious condition called Covid-19 is brought on by the coronavirus SARS-CoV-2. This disease causes a pandemic because this disease spreads very quickly throughout the world. Infected people may have mild symptoms, such as fever, cough, and difficulty breathing. In some cases, Covid-19 sufferers are asymptomatic. Symptoms of diarrhea or an upper respiratory infection (e.g. sneezing, runny nose and sore throat) are less common. Cases can progress to severe pneumonia, multiorgan failure, and death.

The World Health Organization (WHO) and the United States Centers for Disease Control and Prevention estimate that COVID-19 takes between 1 and 14 days to incubate (CDC). Through respiratory tract droplets that are frequently released when sneezing or coughing, the virus is transferred from one person to another. The range of time from virus exposure to the start of clinical symptoms was 1–14 days, with an average of 5 days [1]. A nasopharyngeal swab or sputum sample is used in the reverse-transcription polymerase chain reaction (rRT-PCR) assay, which provides results in a matter of hours to two days. The findings of antibody testing conducted on blood serum samples can be obtained in a matter of days. A computed tomography scan of the chest that reveals

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pneumonia symptoms can also be used to detect infection in addition to symptoms, risk factors, and other tests.

Indonesia has been identified as of the 13th of December 2021, Coronavirus Cases:4.253.992, Deaths:143.753, Recovered:4.102.323 [2]. Figure 1 shows the total Covid-19 Cases in Indonesia, total Covid-19 Deaths in Indonesia, total Covid-19 Currently Infected in Indonesia. So far, the number of Covid-19 cases in Indonesia has decreased in line with the government's policy on PPKM and vaccination. However, the public is still advised to practice clean living such as washing hands with soap, avoiding coughing individuals and refraining from touching their faces with unwashed hands. These are the suggested precautions to stave off this illness. When coughing, it is advised to cover the nose and mouth with a tissue or your elbow bent. Both the World Health Organization (WHO) and the US Centers for Disease Control and Prevention (CDC) advise persons who believe they may have been exposed to an infection should wear a surgical mask and phone a doctor for guidance rather than immediately attending a clinic. Masks are also advised for people providing care for someone who may be ill, however they should not be worn by members of the public, so that the Covid-19 case in Indonesia does not spike again. The emergence of new variants must also be watched out for because this pandemic is not over yet.

In phenomena like this, mathematics can make a contribution. Mathematical modelling has been widely applied to various cases in everyday life, one of which is in the field of epidemiology. Mathematics has a very important role in studying the dynamics of a disease outbreak, starting from the study of source search, distribution, pattern prediction, to handling strategies. This field of study is commonly referred to as epidemiological mathematics. Many studies have been conducted in studying the mathematics of epidemiology. Mathematical modelling of the spread of infectious diseases such as HIV-Aids, diabetes, tuberculosis, dengue fever has been carried out by [3–8]. Furthermore, after the covid-19 pandemic, many researchers have done mathematical modelling of the spread of this disease [9–16]. One of them is the SEIR mathematical model of the spread of covid-19 in Indonesia [17], where in this study the formation of models, analysis and model simulations were carried out. However, optimal control was not given to the mathematical model, so in this study the establishment of the Covid-19 SEIR model with optimal control, model analysis and simulation of the model without control and control was carried out.

The remainder of this work is arranged in the following manner. The SEIR epidemic model is described first, followed by the SEIR epidemic model with optimal control in Sect. 2. The discussion and results are discussed in Sect. 3. The final section summarizes the findings of this study.

2 Mathematical Model

2.1 SEIR Epidemic Model

The SEIR numerical demonstrating on the spread of COVID-19 is a hypothetical report. The strategy used to develop the model is the SEIR model [18] by considering immunization and detachment factors as model boundaries, the model investigation utilizes the

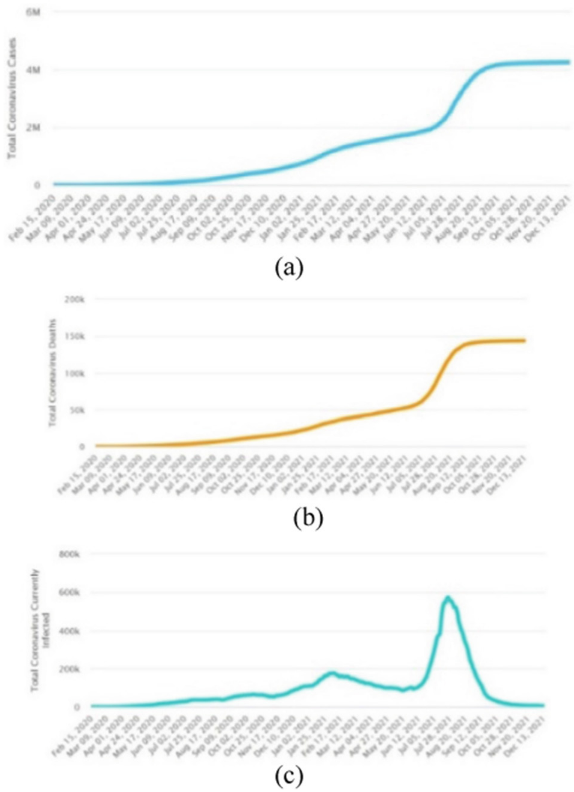

Fig. 1. (a) Total Covid-19 Cases in Indonesia, (b) Total Covid-19 Deaths in Indonesia, (c) Total Covid-19 Currently Infected in Indonesia

age lattice technique [19] to acquire the fundamental generation number and the world-wide soundness for COVID-19 spreading. Mathematical reenactment of model utilize auxiliary information on the quantity of COVID-19 cases in Indonesia [2] by utilizing Mathematica programming to foresee the quantity of COVID-19 cases in Indonesia as an action to forestall the quantity of COVID-19 cases in Indonesia. The SEIR model on the spread of COVID-19 is partitioned into four compartments as follow:

$$dS = \Lambda - (\sigma I + \Lambda + \varphi)S \tag{1}$$

$$dE = \sigma IS - (\gamma + \Lambda)E \tag{2}$$

$$dI = \gamma E - (\Lambda_i + \eta + \Lambda)I \tag{3}$$

$$dR = \eta I + \varphi S - \Lambda R \tag{4}$$

$S(t)$, $E(t)$, $I(t)$, and $R(t)$ are non-negative state variables in the nonlinear SEIR epidemiological model, which defined for Suspected, Exposed, Infected and Recovered.

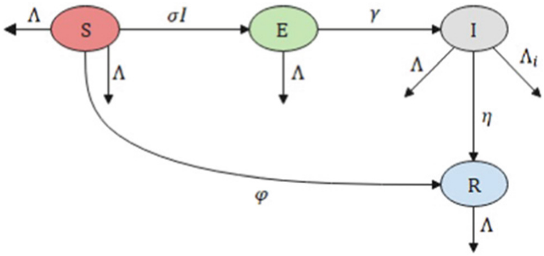

Fig. 2. The flow diagram of the dynamic SEIR model with optimal control

$S(t)$ represents the number of people who are vulnerable to infection (i.e., they have not been infected). When susceptible individuals get infected (i.e., exposed to infection), they are added to the group of exposed persons ($E(t)$), which represents the total number of people who have been exposed to the infection (they are infected and can transmit the virus). Furthermore, by delivering the vaccine to the vulnerable population, and the potential for infected individuals to recover is denoted as $R(t)$.

2.2 SEIR Epidemic Model with Optimal Control

The virus vaccinations that we have are still have a chance of getting infected and many people who died were already in bad health because recovery depends on immune system power. Hence, in this condition the only strategy to stop the virus from spreading right now is to confine persons who are susceptible to infection. Thus, in this section, optimal control theory is used to eradicate epidemics such as decreasing the population of infected individuals and increasing the population of recovered individuals by quarantining susceptible individuals and treating infected individuals with antivyrus drugs. The flow diagram of the dynamic SEIR model with optimal control is shown in Fig. 2. The controller for the dynamic model is provided by:

$$dS = \Lambda - (\sigma I + \Lambda + \varphi + U)S \tag{5}$$

$$dE = \sigma IS + US - (\gamma + \Lambda)E \tag{6}$$

$$dI = \gamma E - (\Lambda_i + \eta + \Lambda + V)I \tag{7}$$

$$dR = \eta I + \varphi S + VI - \Lambda R \tag{8}$$

The initial conditions are non-negative, and the state variables are also non-negative. The major goal in this research is to reduce the function provided by:

$$J = \int_0^T \left(A_1 S + A_2 E + A_3 I + \frac{A_4}{2} U^2 + \frac{A_5}{2} V^2 \right) dt \tag{9}$$

The theory of Pontryagin Maximum Principle [20, 21], provides the required requirements that an optimal control must fulfill. The systems (5)–(9) are transformed into a problem of minimizing a Hamiltonian H pointwise with regard to the controls ui using this principle. To proceed, we first write the Hamiltonian H, which is given by:

$$\begin{aligned} H = A_1 S + A_2 E + A_3 I + \frac{A_4}{2} U^2 + \frac{A_5}{2} V^2 \\ + \psi_1(\Lambda - (\sigma I + \Lambda + \varphi + U) S) \\ + \psi_2(\sigma IS + US - (\gamma + \Lambda) E) \\ + \psi_3(\gamma E - (\Lambda_i + \eta + \Lambda + V) I) \\ + \psi_4(\eta I + \varphi S + VI - \Lambda R) \end{aligned} \tag{10}$$

The adjoint variables related with the state variables S, E, I, and R are denoted by i for i = 1, 2, 3, 4. The proper partial derivatives of H with respect to the state variables can be used to derive the system of adjoint equations.

Theorem. There exists adjoint variables $\psi_1(t), \psi_2(t), \psi_3(t), \psi_4(t)$ with the corresponding system of the solutions $S^*(t), E^*(t), I^*(t), R^*(t)$ and optimal controls $(U^*(t), V^*(t))$, that fulfill

$$\dot{\psi_1}(t) = -\frac{\partial H}{\partial S(t)} = -(A_1 - \psi_1(t)(\sigma I(t) + \Lambda + \varphi + U(t)) + \psi_2(t)(\sigma I(t) + U(t)) + \psi_4(t)\varphi) \tag{11}$$

$$\dot{\psi_2}(t) = -\frac{\partial H}{\partial E(t)} = -(A_2 - \psi_2(t)(\gamma + \Lambda) + \psi_3(t)\gamma \tag{12}$$

$$\begin{aligned} \dot{\psi_3}(t) = -\frac{\partial H}{\partial I(t)} = -\,(A_3 - \psi_1(t)\sigma S(t) + \psi_2(t)\sigma S(t) \\ -\psi_3(t)(\Lambda_i + \eta + \Lambda + V(t)) + \psi_4(t)(\eta + V(t))) \end{aligned} \tag{13}$$

$$\dot{\psi_4}(t) = -\frac{\partial H}{\partial R(t)} = -(-\psi_4(t)\Lambda) \tag{14}$$

We can solve the optimum controls utilizing the optimality conditions $\nabla^H_{U(t)} = 0$ and $\nabla^H_{V(t)} = 0$. Hence,

$$U^*(t) = \frac{\psi_1(t)S^*(t) - \psi_2(t)S^*(t)}{A_4} \tag{15}$$

$$V^*(t) = \frac{\psi_3(t)I^*(t) - \psi_4(t)I^*(t)}{A_5} \tag{16}$$

$$\text{therefore, } U^*(t) = \begin{cases} 0, & \frac{\psi_1(t)S^*(t)-\psi_2(t)S^*(t)}{A_4} \geq 1 \\ \frac{\psi_1(t)S^*(t)-\psi_2(t)S^*(t)}{A_4}, & 0 < \frac{\psi_1(t)S^*(t)-\psi_2(t)S^*(t)}{A_4} < 1 \\ 1, & \frac{\psi_1(t)S^*(t)-\psi_2(t)S^*(t)}{A_4} \leq 0 \end{cases} \tag{17}$$

$$V^*(t) = \begin{cases} 0, & \frac{\psi_3(t)I^*(t)-\psi_4(t)I^*(t)}{A_5} \geq 1 \\ \frac{\psi_3(t)I^*(t)-\psi_4(t)I^*(t)}{A_5}, & 0 < \frac{\psi_3(t)I^*(t)-\psi_4(t)I^*(t)}{A_5} < 1 \\ 1, & \frac{\psi_3(t)I^*(t)-\psi_4(t)I^*(t)}{A_5} \leq 0 \end{cases} \tag{18}$$

so that the optimal control that has been obtained, can be formed as follows:

$$U^*(t) = min\left(max\left(0, \frac{\psi_1(t)S^*(t) - \psi_2(t)S^*(t)}{A_4}\right), 1\right) \tag{19}$$

$$V^*(t) = min\left(max\left(0, \frac{\psi_3(t)I^*(t) - \psi_4(t)I^*(t)}{A_5}\right), 1\right) \tag{20}$$

Subsequently, by applying utilizing the optimal control [22, 23], the accompanying optimality framework to optimal control is acquired as:

$$\dot{S}^*(t) = \Lambda - \left(\sigma I(t) + \Lambda + \varphi + min\left(max\left(0, \frac{\psi_1(t)S^*(t) - \psi_2(t)S^*(t)}{A_4}\right), 1\right)\right)S(t) \tag{21}$$

$$\dot{E}^*(t) = \sigma I(t)S(t) + min\left(max\left(0, \frac{\psi_1(t)S^*(t) - \psi_2(t)S^*(t)}{A_4}\right), 1\right)S(t) - (\gamma + \Lambda)E(t) \tag{22}$$

$$\dot{I}^*(t) = \gamma E(t) - \left(\Lambda_i + \eta + \Lambda + min\left(max\left(0, \frac{\psi_3(t)I^*(t) - \psi_4(t)I^*(t)}{A_5}\right), 1\right)\right)I(t) \tag{23}$$

$$\dot{R}^*(t) = \eta I(t) + \varphi S(t) + min\left(max\left(0, \frac{\psi_3(t)I^*(t) - \psi_4(t)I^*(t)}{A_5}\right), 1\right)I(t) - \Lambda R(t) \tag{24}$$

The adjoint equation are written as:

$$\begin{aligned} \dot{\psi}_1(t) = -\frac{\partial H}{\partial S(t)} = &-\big(A_1 - \psi_1(t)\big(\sigma I(t) + \Lambda + \varphi + U^*(t)\big) \\ &+\psi_2(t)\big(\sigma I(t) + U^*(t)\big) + \psi_4(t)\varphi\big) \end{aligned} \tag{25}$$

$$\dot{\psi}_2(t) = -\frac{\partial H}{\partial E(t)} = -(A_2 - \psi_2(t)(\gamma + \Lambda) + \psi_3(t)\gamma) \tag{26}$$

$$\begin{aligned} \dot{\psi}_3(t) = -\frac{\partial H}{\partial I(t)} = &- (A_3 - \psi_1(t)\sigma S(t) + \psi_2(t)\sigma S(t) \\ &- \psi_3(t)\big(\Lambda_i + \eta + \Lambda + V^*(t)\big) + \psi_4(t)\big(\eta + V^*(t)\big) \end{aligned} \tag{27}$$

$$\dot{\psi}_4\ (t) = -\frac{\partial H}{\partial R(t)} = -(-\psi_4(t)\Lambda) \tag{28}$$

3 Results and Discussion

To support the theoretical findings reported in the section before, numerical results are offered in this section. The parameter values of the SEIR mathematical model are shown in Table 1.

The mathematical model that has been developed in Eqs. (5–8) is simulated using the Pontryagin Maximum Principal scheme utilizing the numerical approach Runge Kutta order 4. The graphical results of SEIR Epidemic model without control are obtained in Fig. 3(a). The parameter values presented in Table 1 have produced graphic results for the optimal control problem compared with the system that without control. The outcomes show it to be true of the graph given that the control is much more effective for the elimination of the disease. Figure 3(b) shows the dynamics of the suspected class before and after being given optimal control. It can be seen that before optimal control was carried out, suspected individuals experienced a decrease but not so significantly. It shown a very considerable drop since the first day after receiving optimal control and was close to zero until the last day of the simulation. The dynamics of individual exposed, shown in Fig. 3(c), shows that before being given control and after being given control there was a significant decrease. Figure 3(d) represents the dynamics of infected individuals. The figure shows that before optimal control was given, infected individuals on the first day experienced an increase which on the next day decreased but decreased slowly and not so significantly. In contrast to after being given control, infected individuals decreased drastically and went to zero. Individuals who have recovered are represented in Fig. 3(e). Before being given control, individuals who recovered experienced an increase from the first day but not significantly. Meanwhile, after being given control A considerable increase was seen on the first day, and the next day it decreased but the decrease was not significant and on the last day the simulation was still greater than before being given control. So it can be concluded that the control works well on the system used.

It can be shown that the control strategy used in this numerical simulation works well to eliminate the diseased population while increasing the number of vacillating and recovering people. Corona virus infections in the community can be decreased through social isolation, mask use, routine hand washing with sanitizer, immunization of individuals, quick diagnosis, and the potential of early treatment.

Table 1. Parameter Value

Parameter	Estimated Value
Λ	6.25×10^{-3}
σ	0.62×10^{-8}
η	0.0006667
Λ_i	7.344×10^{-7}
φ	1%
γ	3

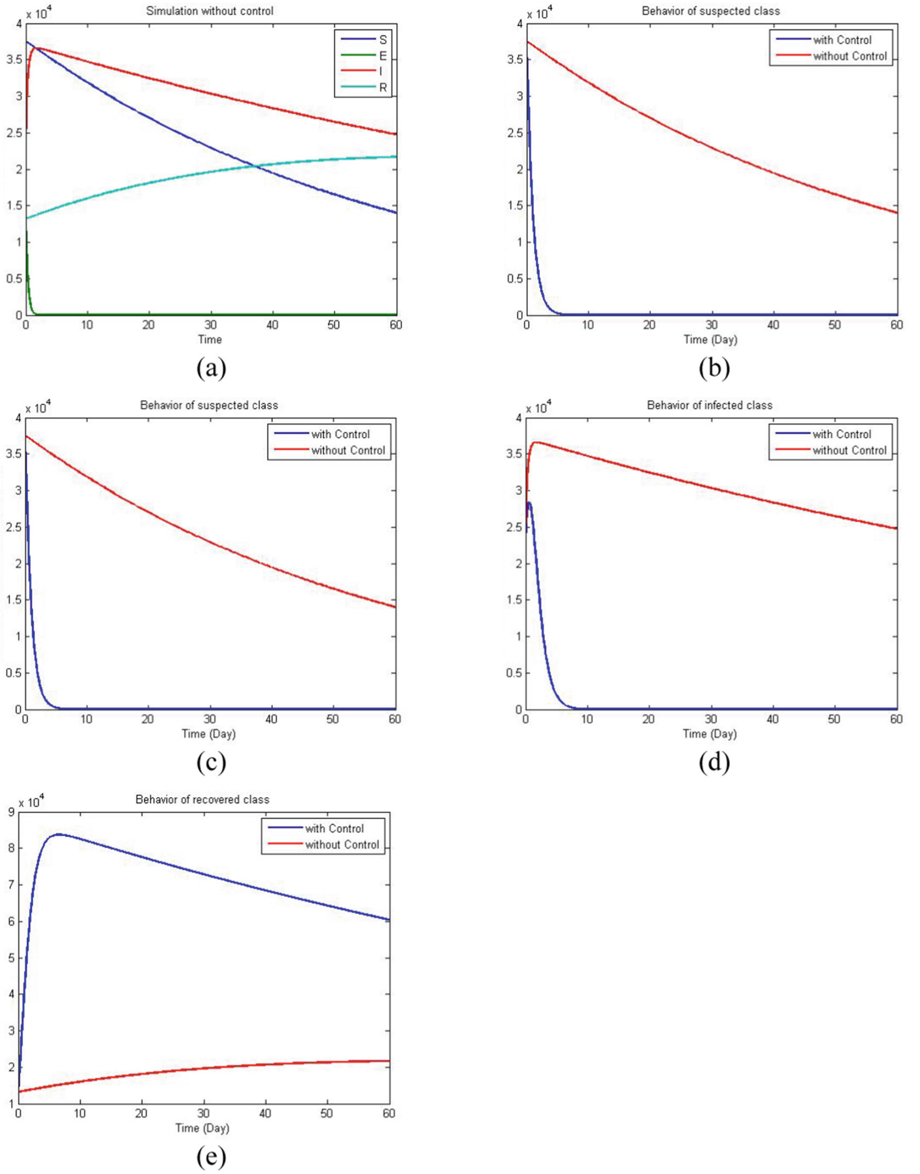

Fig. 3. (a) SEIR Epidemic Model without Control, (b) Suspected Class, (c) Exposed Class, (d) Infected Class, (e) Recovered Class

As it is known that the COVID-19 infection has caused huge losses to human society and many developing countries are facing huge financial losses for their economies. Therefore, proper individual vaccination and individual prevention from for least developed nations to retain their people and economies, preventing infection should be a top priority.

4 Conclusion

Based on the results of the study, using optimal control theory, a mathematical model for coronavirus infection was created. This mathematical model of the spread of COVID-19 is divided into four parts, SEIR. For each disease in the human population it is necessary to control it with available measures that researchers and professionals engaged in the study, treatment, and prevention of the disease have recommended. Considering the possibility of control that is considered effective, an effective control model has been put into a model that has been formulated. There are two optimal control theories proposed in this study, namely treatment control and quarantine. The optimal control system produced in this study can solve the optimal control problem and achieve the desired results. The initial conditions used for each state are identified using actual Indonesian data. The simulation is done using matlab software which is represented in a graph and shows the controls used to reduce infection better.

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